International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249–6890; ISSN (E): 2249–8001
Vol. 10, Issue 2, Apr 2020, 1167–1178

EFFECT OF RADIATION PRESSURE ON RESONANT PERIODIC ORBITS IN PHOTO GRAVITATIONAL RESTRICTED THREE-BODY PROBLEM

PRASHANT KUMAR & Dr. RAM KRISHNAN SHARMA

Aerospace Engineering, Sandip University, Nashik, India

Karunya Institute of Technology and Sciences, Coimbatore, India

ABSTRACT

A study of resonant periodic orbits is made in the photogravitational restricted problem of three bodies (PRTBP). Sun-Jupiter and Sun–Uranus systems are chosen and Poincaré surface of sections (PSS) is utilized in this study. Using the technique of PSS, locations of the resonant periodic orbits (4:3 and 3:2) are determined and solar radiation pressure effect is studied on these periodic orbits. Solar radiation pressure helps in merging these resonant periodic orbits (4:3 and 3.2 into 1:1) and it was found that merging takes place at almost the same location for Jacobi constant C = 2.9 for both the systems. By increasing radiation pressure, we have located the merging location of resonant periodic orbits at different values of C. At the time of merging, these orbits become near-circular. The period and size of these orbits reduce with the increase in radiation force.

KEYWORDS: Photogravitional problem of three bodies, Resonance, Poincaré surface of sections & Sun radiation force

Received: Feb 22, 2020; **Accepted**: Mar 12, 2020; **Published**: Apr 07, 2020; **Paper Id.**: IJMPERDAPR2020113

Original Article

1. INTRODUCTION

Restricted problem of three bodies (RTBP) plays very significant role in celestial mechanics and space science. The study of the motion of the third body which has infinitesimal mass and does not change the motion of the two masses, which are known as primaries and they are lying in the same plane is known as RTBP. Many great mathematicians e.g. Euler, Lagrange, Jacobi, Hill, Hamilton etc. have worked extensively on RTBP and made significant contributions. The book of Szebehely [1] provides systematic coverage of the literature on the subject as well as derivations of some of the important results. The equations of motion having non linearity, the analytical solutions do not provide very good insight into the solution. PSS has been utilized in the past by many researchers to analyze different kinds of orbits. Kolmogorov-Arnold-Moser (KAM) theoremsaysthata fixed point on the PSS provides a periodic orbit.

Poynting [2] and Robertson [3] established that the effect produced by Sun pressure depends on its geometry and other characteristics. Radzievskii[4] provided a simplified theory. After that many significant research in the PRTBP are by Chernikov [5], Perezhogin [6], Bhatnagar and Chawla [7], Schuerman [8], Simmons et al. [9], Roman [10], Kushvah and Ishwar [11] and Das et al. [12]. Sharma [13] included the flattening of the bigger and small primaries, respectively, in the photo-gravitational problem. Further, Dutt and Sharma [14] studied solar radiation force effect on periodic orbits in Sun-Mars system.

The effect of resonance is significant in understanding the characteristics of dynamical systems. The earlier works on the resonance in the dynamic evolution of the solar system are by Roy and Ovenden [15]. Useful review of the theory of resonance have been given by Greenberg [16]and Peale[17]. Murray and Dermot [18] made a detailed

research in the theory of the resonance. Quarle, Musielak and Cuntz [19] studied the mean-motion resonance for the coplanar CRTBP for different mass ratios and recently Wang and Malhotra [20] studied the mean motion resonance in the CRTBP for larger values of eccentricity. In this paper, we study the restricted three body problem when the more massive primary is a source of radiation with its equatorial plane coincident with the plane of motion. The more massive primary is the Sun and smaller primary is the Jupiter or Uranus. The method of PSS is used to describe the nature and locations of periodic orbits.

The periodic orbits around Sun with 4:3 and 3:2 first-order interior resonances in Sun-Jupiter and Sun-Uranus PRTBP are studied. Jupiter has asteroids having first-order mean motion resonances with it. They hold significant information regarding the process of giant planets [Brož and Vokrouhlicky, 21]. It is known that the population of the asteroids exist in the Jovian first-order mean motion resonance 3:2 (Hilda group) and 4:3(Thule group). The authors main results were an update of the observed 3:2 and 4:3 resonant populations; discovery of two new objects in the 4:3 resonance and description of two asteroid families located inside the 3:2 group. A study was initiated by Nishanth and Sharma [22] in the PRTBP to find the effect of the increase in Jacobian constant C on the above orbits to transform to resonance with 1:1. In this paper, we have carried out a detailed study in PRTBP to find the effect of solar radiation force (ε) and Jacobian constant C on these obits in the Sun-Jupiter RTBP. It is found that both these parameters help in merging these orbits (4:3 and 3.2 into 1:1 resonance). We have located the merging points for different values of C. During the time of merging, these orbits become near-circular. The period and size of these orbits reduce when Sun radiation force increases.

2. PLANAR PHOTOGRAVITIONAL RESTRICTED PROBLEM

We consider Sun as the source of radiation. In terms of dimensionless synodic coordinate system with origin at the center of mass of the two bodies, whose locations are at (μ,0) and (μ-1,0) with equatorial plane same as the plane of motion, where $\mu = m_1/(m_1+m_2) \leq 1/2$, (=0.0009537284) m_1 and m_2 are masses of the Sun and Jupiter, respectively. Radiation pressure effect is written as mass reduction factor q =1 - ε, the radiation coefficient $\varepsilon = F_p/F_g$, where F_p is a force caused by radiation and F_g which results from gravitation. q is expressed in terms of particle radius 'a', density 'δ' and radiation pressure efficiency 'χ' (in CGS system) as

$$q = 1 - \frac{5.6 \times 10}{a\delta}, \qquad (2.1)$$

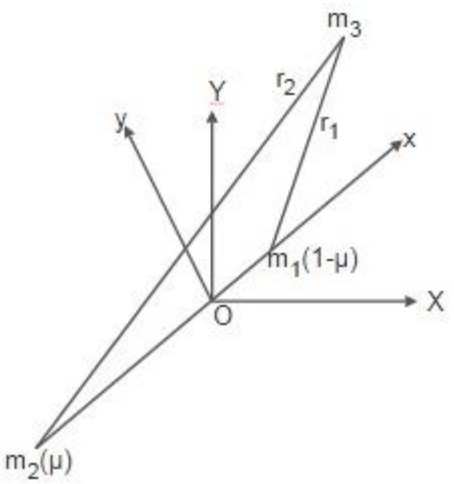

Figure 1: The Rotating (synodic) Coordinate System

If we know the mass and the luminosity of the radiating body, we can find ε for known radius and density. $F_p$changes with distance following the same law of gravitational attraction. And it acts opposite to it. It is possible to consider that the result of action of this force will lead to reducing the effective mass of the Sun or a particle. Since the effect of reducing the mass of a particle depends upon its properties, it is acceptable to speak about a reduced mass of the particle. Sun's resulting force, which acts on the particle is (Sharma [23]; Kalvouridis et al. [24])

$$F = F_g – F_p = (1 – F_p / F_g)\, F_g = (q)\, F_g, \tag{2.2}$$

For q =1, there is no radiation effect, and for $0 < q \le 1$, gravitational force exceeds radiation and we consider this case for our detailed study.

The planar equations of motion of the third body are (Bhatnagar and Chawla [7])

$$\ddot{x} - 2\dot{y} =, \tag{2.3}$$

$$\ddot{y} + 2\,\dot{x} =, \tag{2.4}$$

Where,

$$\Omega = \frac{1}{2}[(1-\mu)r_1{}^2 + \mu\, r_2^2] + \frac{q\,(1-\mu)}{r_1}. \tag{2.5}$$

$$r_1^2 = (x-\mu)^2 + y^2,$$

$$r_2^2 = (x+1-\mu)^2 + y^2,$$

The Jacobi integral is

$$\dot{x}^2 + \dot{y}^2 = 2\,\Omega - \tag{2.6}$$

3. POINCARÉ SURFACE OF SECTION

Periodic orbits around Sun in the Sun-Jupiter and Sun–Uranus system are studied using PSS method. We plot the values of x and in the three dimensional space by defining a plane with y=0. We consider a fixed value of the Jacobi constant C and whenever > 0, we notice that 3D space reduces to 2D space. PSS is a good method to find regular and chaotic orbits. The smooth and well defined island define regular trajectory, which can be periodic represented by a point.

For each C, numerical integration is done. The initial conditions are selected as follows: x is selected so that y = = 0 and > 0. In order to generate the PSS, the equation of motion (2.3) and (2.4) are integrated with the help of fourth-order RungeKutta method with Gill modification with integration step size Δt of 0.0005. PSS contains discrete points of trajectories.

4. EVOLUTION OF 3:2 AND 4:3 RESONANT PERIODIC ORBITS IN THE SUN-JUPITER SYSTEM

In the present study, we have generated the trajectories of the third particle for different initial conditions for the Sun-Jupiter system by considering Sun as source of radiation. We have generated the trajectories at various locations for C = 2.95 and q = 1 given in Figure 2. The trajectory of the particle with starting value x_0=0.4990, y_0 = 0.0, = 0.0 and $_0$ > 0.0 is determined from Eq. (2.6) and this trajectory is close to 3:2 resonance. The trajectory for C = 2.95 at x = 0.5885 is close to

4:3 interior resonance.

To study Sun's radiation force effect, we take the orbit having resonance 3:2 as shown in Figure 2 for C = 2.95 and ε = 1- q = 1- 0.99 = 0.01 at x = 0.5221. The transformation of this periodic orbit with 3:2 interior resonances is shown in Figures 4 to 6 by increasing the radiation pressure ε from 0.01 to 0.04. It is also observed that the period of time of these orbits decreases as Sun's radiation force increases. Table 1 provides the initial locations of these orbits for Ɛ = 0.01 to 0.07. It may be noted that the orbits move towards Jupiter when Sun's radiation force increases.

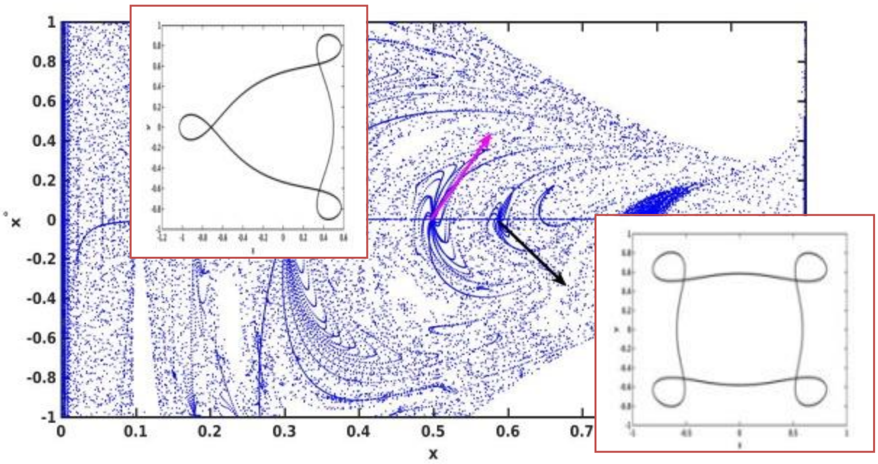

Figure 2: Surface of Section of Poincaré with C = 2.95 and ε = 0

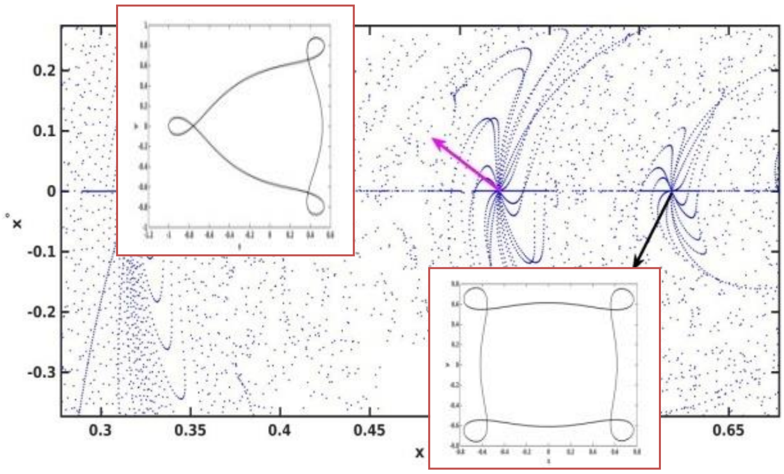

Figure 3: Surface of Section of Poincaré with C = 2.95, ε = 0.01

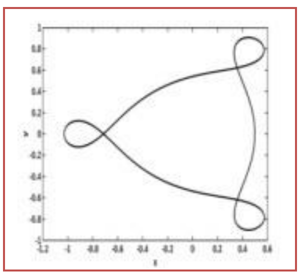

Figure 4: C = 2.95, ε= 0.01, x= 0.5221

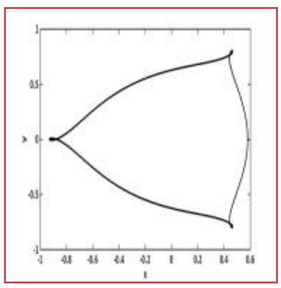

Figure 5: C = 2.95, ε=0.03, x = 0.5632

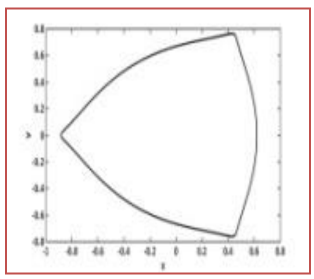

Figure 6: C = 2.95, ε = 0.04x = 0.5720

Table 1: Initial Location of the Orbits

q	ε	Resonance Periodic Orbit
		3:2
1	0	0.4990
0.99	0.01	0.5221
S0.95	0.05	0.5830
0.93	0.07	0.5841

5. MERGING OF 3:2 AND 4:3 RESONANT PERIODIC ORBITS

The Surface of section of Poincaré is employed to find 3:2 and 4:3interior resonance periodic orbits around Sun under Sun's radiation force. Δx and Δt in the numerical integration were suitably selected. PSS were generated for C = 2.9 with and without the radiation pressure and presented in Figures 7 and 8. These orbits starting between x = 0.4 and x = 0.55 are shown in Figure 7. These orbits shift towards Jupiter with increase in radiation pressure (ε). Their shape changes gradually to elliptic orbits. It is observed that with increase in ε, 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.7016 and the resonance of the merged periodic orbit becomes 1:1. Table 2 provides period of the orbits for ε = 0 to 0.09. It may be seen from Figures 6 and 9 that as ε increases from 0.01 to 0.09, the orbit becomes gradually circular and smaller in size.

Similarly, we have also obtained PSS for Sun-Jupiter RTBP for C = 3.0. In this case 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.7237 and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun with further increase in radiation pressure. Table 3 provides a comparison of the time period of the periodic orbits for ε = 0 to 0.03. It may be noted from Figures 12 to 13 that with increase in ε from 0.0 to 0.03, the orbit becomes gradually circular and smaller in size.

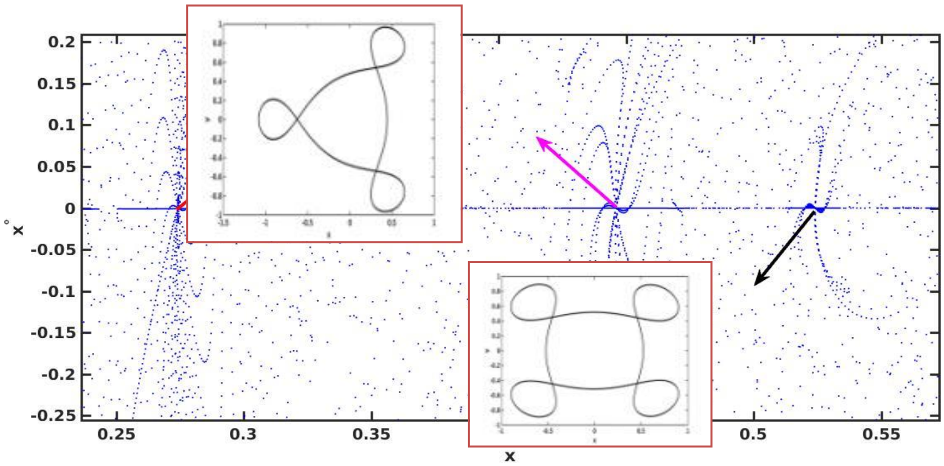

Figure 7: Surface of section of Poincaré with C =2.9 and ε= 0

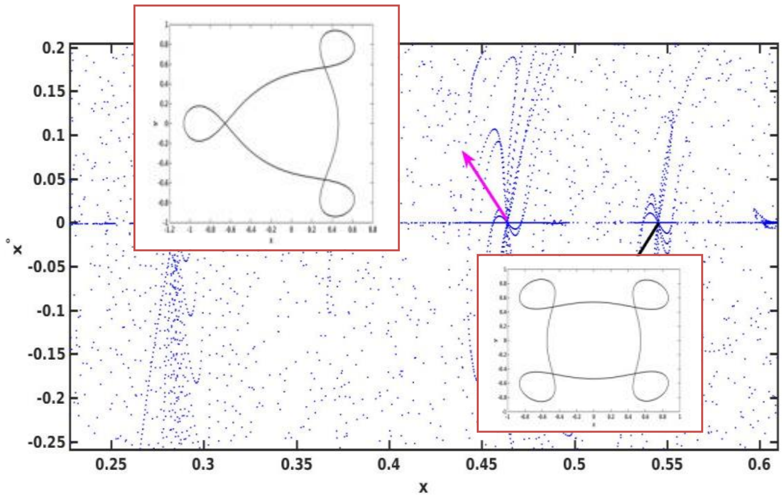

Figure 8: Surface of section of Poincaré with C =2.9 and ε = 0.01

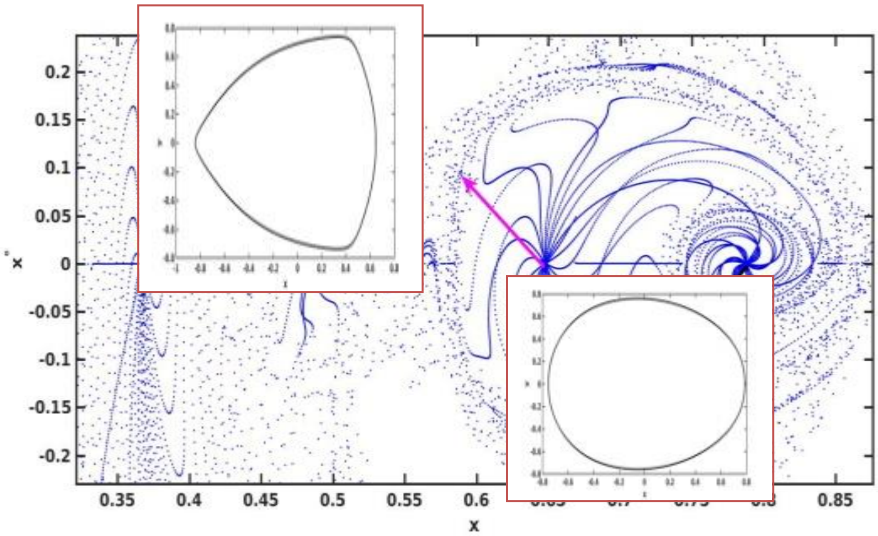

Figure 9: Surface of Section of Poincaré with C =2.9 and ε = 0.07

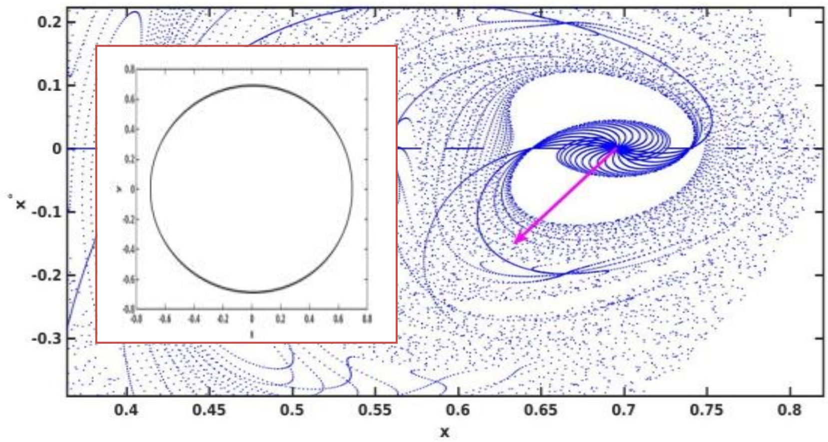

Figure 10: Surface of Section of Poincaré with C =2.9 and ε = 0.09

Table 2: Initial location and time period of the orbit

q	ε	Resonance			
		3:2		4:3	
		Location	Period	Location	Period
1	0	0.4460	12.5639	0.4318	18.8596
0.99	0.01	0.4630	12.5450	0.4450	18.8545
0.97	0.03	0.45054	12.5430	0.4930	18.8276
0.95	0.05	0.5606	12.5421	0.5589	18.7813
0.93	0.07	0.6972	12.5400	0.6690	14.8860
0.91	0.09		9.7437	Merging	

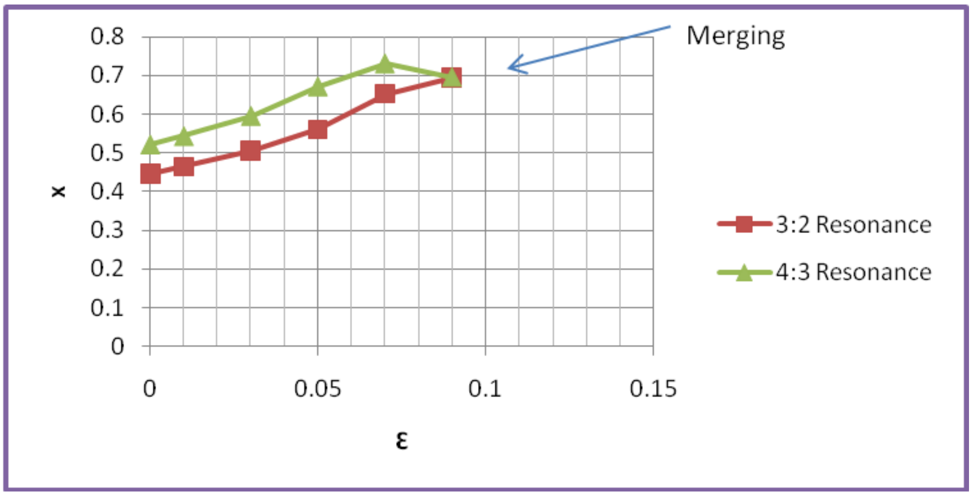

Figure 11: Merging location of 3:2 and 4:3 period orbits for C=2.9

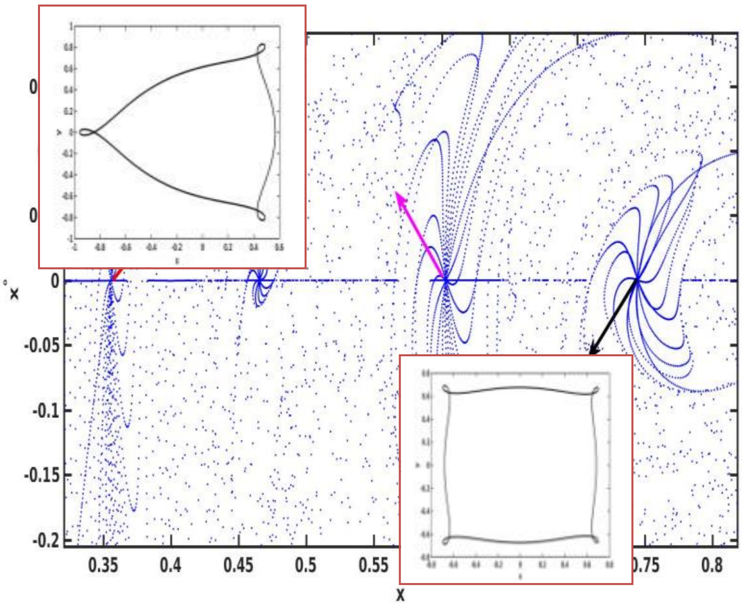

Figure 12: Surface of Section of Poincaré with C = 3.0 and ε = 0.01

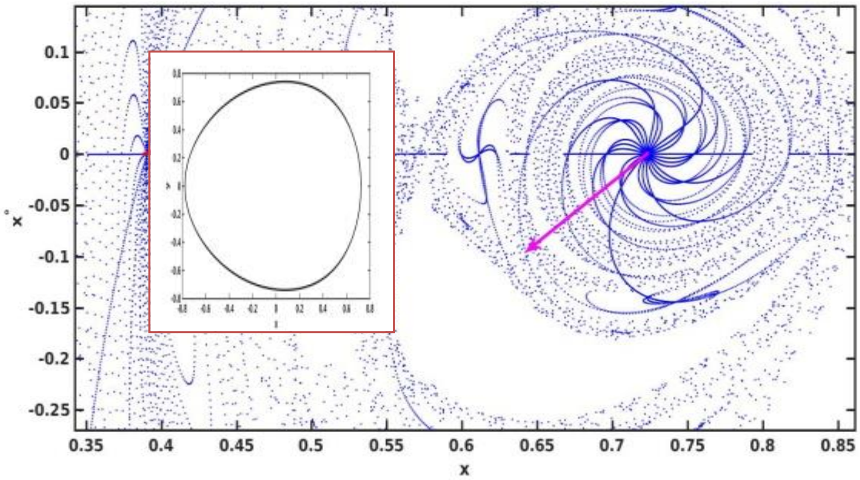

Figure 13: Surface of section of Poincaré with C =3.0 and q= 0.03

Table 3: Initial Location and Time Period of the Orbits

q	Ɛ	Resonance			
		3:2		4:3	
		Location	Period	Location	Period
1	0	0.5683	12.5451	0.6830	18.7712
0.99	0.01	0.6032	12.5181	0.7449	18.5835
0.98	0.02	0.6504	12.4725	0.8036	15.513
0.97	0.03	0.7237	11.9886	Merging	

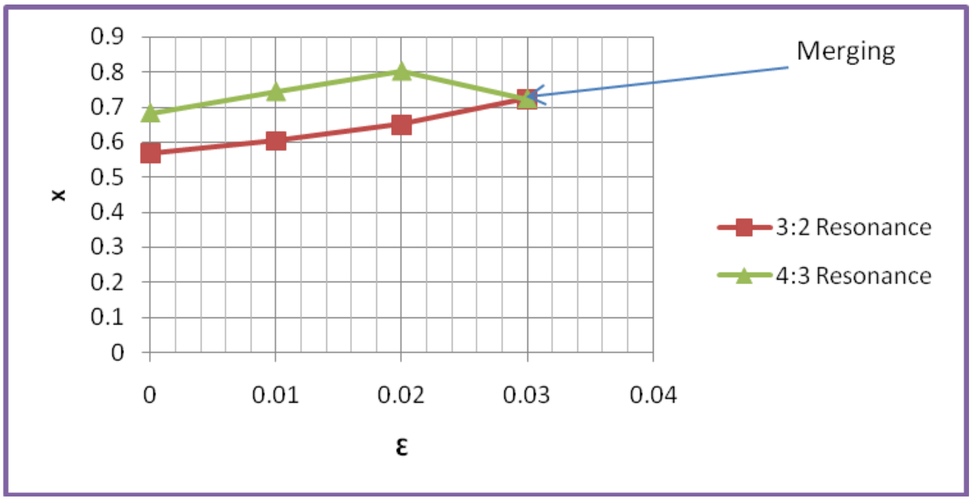

Figure 14: Merging Location of 3:2 and 4:3 Period Orbits for C = 3.0

EVOLUTION OF PERIODIC ORBIT IN SUN-URANUS SYSTEM

To study the effect of the mass parameter, we have obtained PSS for the Sun-Uranus system whose mass ratio is very small for one value of C=2.9. In this case 4:3 resonant periodic orbit merges with 3:2 resonant periodic orbit at x = 0.6949 and the resonance of the merged periodic orbit becomes 1:1. This orbit shifts towards Sun as its radiation force increases. Table 4 gives the location of periodic orbit for Ɛ = 0 to 0.09. It is interesting to note that merging is found at almost the same location as obtained in the Sun-Jupiter for C = 2.9.

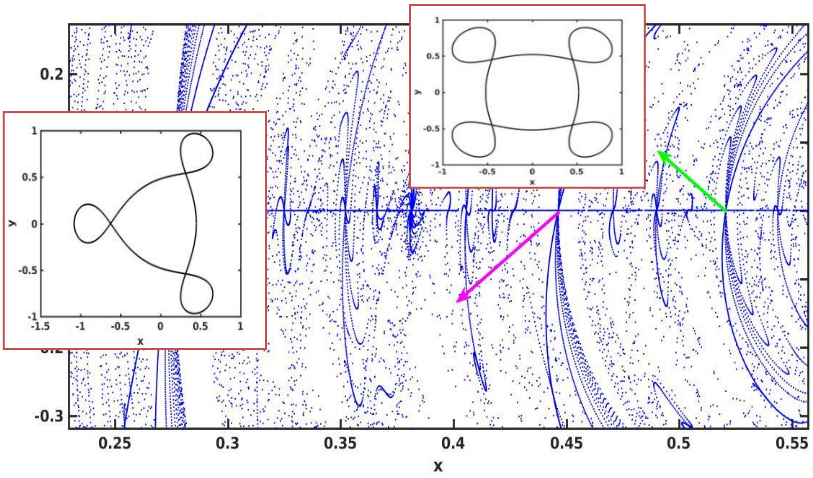

Figure 15: Surface of Section of Poincaré with C =2.9 and ε =0

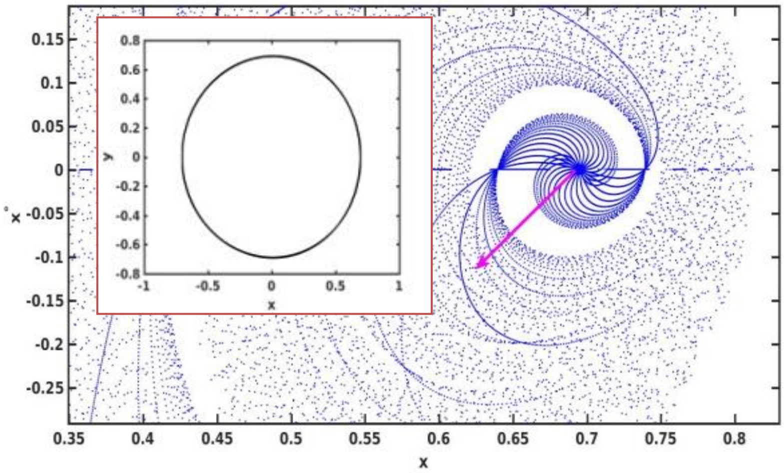

Figure 16: Surface of Section of Poincaré with C =2.9 and ε =0.09

Table 4: Initial Location and Time Period of the Orbits

q	ε	Resonance			
		3:2		4:3	
		Location	Period	Location	Period
1	0	0.4464	12.5610	0.5207	18.8400
0.95	0.05	0.5606	12.5500	0.6705	18.8200
0.91	0.09	0.6949	9.7000	Merging	

CONCLUSIONS

The effect of Sun's radiation force is studied on the resonant periodic orbits with 4:3 and 3:2 first-order interior resonances for Sun–Jupiter and Sun–Uranus PRTBP. The merging of these resonant periodic orbits into 1:1 are found with the help of surface of section of Poincaré. The period and size of orbits decrease with increase in Sun's radiation force. The radiation force decreases with the increase in Jacobian constant for merging of the orbits.

REFERENCES

1. *Szebehely V. Theory of Orbits. USA: Academic Press, 1967.*
2. *J. H. Poynting, "Radiation in the Solar System: Its Effect on Temperature and Its Pressure on Small Bodies,"Philosophical Transactions of the Royal Society of London A, vol. 202, pp. 525-552, 1903.*
3. *H. Robertson, "Dynamical effects of radiation in the solar system,"MonthlyNoties of the Royal Astronomical Society, vol. 97, pp. 423-437,1937.*
4. *RadzievskiiV. V., "The Restricted Problem of Three-Body Taking Account of Light Pressure," Astronomicheskii-Zhurnal, vol. 27, pp. 250-256, 1950.*
5. *Yu. A. Chernikov, "The photogravitational restricted three-body problem," Soviet Astronomy-AJ., vol. 14, no.1, pp. 176-181,1970.*
6. *A. A. Perezhogin, "Stability of the sixth and seventh libration points in the photogravitational restricted circular three-body problem," Soviet Astronomy Letters, vol. 2, pp. 172,1976.*
7. *Bhatnagar, K. B. and Chawla, J. M.,"A Study of the Lagrangian Points in the PhotogravitationalRestrictedThree-Body Problem,"Indian Journal of Pure and Applied Mathematics, vol. 10, pp. 1443-1451, 1979.*
8. *D. W. Schuerman, "The restricted three-body problem including radiation pressure", Astrophysical Journal, vol. 238, no. 1, pp. 337-342,1980.*
9. *J. F. L. Simmons, A. J. C. McDonald andJ. C. Brown, "The Restricted Three-Body Problem with Radiation Pressure", Celestial Mechanics, vol. 35, pp. 145-187, 1985.*
10. *R. Roman,"The Restricted Three-Body Problem. Comments on the `Spatial' Equilibrium Points", Astrophysics and Space Science, vol.275, pp. 425-429, 2001.*
11. *[11] Kushvah B.S and IshwarB.,"Triangular equilibrium points in the Generalized Photogravitational Restricted Three Body Problem with Poynting-Robertson Drag," Review Bulletin of the Calcutta Mathematical Society, vol. 12, pp.109-114, 2004.*
12. *M. K. Das, P. Narang, S. Mahajan and M. Yuasa, "Effect of Radiation on the Stability of Equilibrium Points in the Binary Stellar Systems: RW-Monocerotis, Krüger 60,"Astrophysics and Space Science, vol. 314, pp. 261, 2008.*
13. *R. K. Sharma, "On Linear Stability of Triangular Libration Points of the Photogravitational Restricted Three-Body Problem when the More Massive Primary is an Oblate Spheroid," Sun and Planetary System: Proceedings of the Sixth European Regional Meeting in Astronomy, Dubrovnik, Yugoslavia, pp. 435-436,1982.*
14. *Othman, A. A., et al. "Thermal annealing and UV induced effects on the structural and optical properties of capping free ZnS nanoparticles synthesized by Co-precipitation method." International Journal of General Engineering and Technology, 3, 9 16 (2014).*
15. *P. Dutt, and R. K. Sharma, "Evolution of periodic orbits in the Sun-Mars system," Journal of Guidance, Control and Dynamics", vol. 35, pp. 635-644, 2011.*
16. *A. E. Roy and M. W. Ovenden, "On the Occurrence of Commensurable Mean Motions in the Solar System,"Monthly Notices of the Royal Astronomical Society, vol. 114, pp. 232-241, 1954.*
17. *Greenberg, R.: Orbit-orbit resonances among natural satellites. In:Burns, J.A. (ed.) Planetary Satellites. University of Arizona Press, Tucson,1977.*
18. *[17] Peale, S.J.: Orbital resonances, unusual configurations and exotic rotationstates among planetary satellites. In: Burns,*

J.A., Matthews, M.S. (eds.) Satellites. University of Arizona Press, Tucson,1986.

19. *Murray, C.D., Dermot, S.F.: Solar System Dynamics. Cambridge UniversityPress, Cambridge,1999.*
20. *B. Quarles, Z.E. Musielak and M. Cuntz,"Study of resonances for the restricted 3-body problem,"AstronomischeNachrichten, vol. 333, pp. 551-560, 2012.*
21. *X. Wang and Malhotra, R. "Mean Motion Resonances at High Eccentricities: the 2:1 and the 3:2 Interior Resonances" AJ,154, articleid 225,13pp., 2017.*
22. *Prakhya, Rajesh Kumar, and C. Krishna Reddy. "Comparison of Various Types of Rooftop Grid Connected Solar Plants: A Case Study." International Journal of Electrical and Electronics Engineering Research (IJEEER) 7 (2017): 11-22.*
23. *M. Brož and D. Vokrouhlicky, "Asteroid families in the first-order resonances with Jupiter," Monthly Notices ofthe Royal Astronomical Society, vol. 390, pp. 715-732, 2008*
24. *PushparajNand Sharma, R. K.: "Interior Resonance Periodic Orbits in Photogravitational Restricted Three-bodyProblem", Advances in Astrophysics, Vol. 2, No. 1, pp. 263-272, 2017.*
25. *R. K. Sharma, "The linear stability of liberation points of the photogravitational restricted three-body problem when the smaller primary is an oblate spheroid," Astrophysics and Space Science, vol. 135, pp. 271-281,1987.*
26. *Nikam, Harshal, P. R. E. M. Mishra, and Sayali Bharambe. "Design and Analysis of Brake Rotor with Parameter Optimization." International Journal of Automobile Engineering Research and Development (IJAuERD) 4.4 (2014): 21-30.*
27. *Kalvouridis, M. Arribas and A. Elipe, "Parametric evolution of periodic orbits in the restricted four-body problem with radiation pressure,"P*
28. *Shah, Mehul K., Chetan B. Bhatt, and JAIMIN B. Dave. "NIR spectroscopy: Technology ready for food industries applications." International Journal of Applied and Natural Sciences (IJANS) 5 (2016).*

AUTHOR PROFILE

Prashant Kumar

Academic Qualifications:

AMAeSI (Aerodynamics),

M.Tech. (Aeronautical, VTU University),

Ph. D (Pursuing),

Publications: 4 technical papers in International Journals, 2 papers presented in International Conferences.

Area of Specialization: Aerodynamics, CFD, Orbital Mechanics